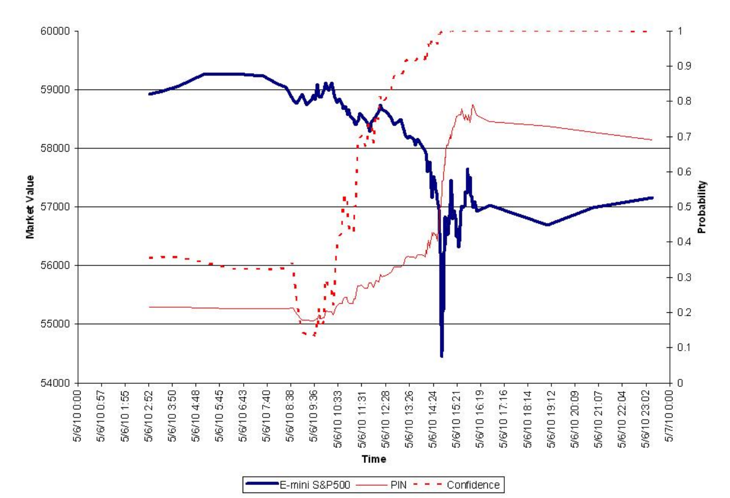
VPIN

The **V**olume Synchronized **P**robability of **IN**formed Trading, commonly known as **VPIN**, is a mathematical model used in financial markets for multiple purposes. Initially proposed by Professors Maureen O'Hara and David Easley, of Cornell University, in cooperation with Marcos Lopez de Prado, of Tudor Investment Corporation and RCC at Harvard University. This model received media attention when it was shown to anticipate the 'flash crash' of May 6, 2010 more than one hour in advance.

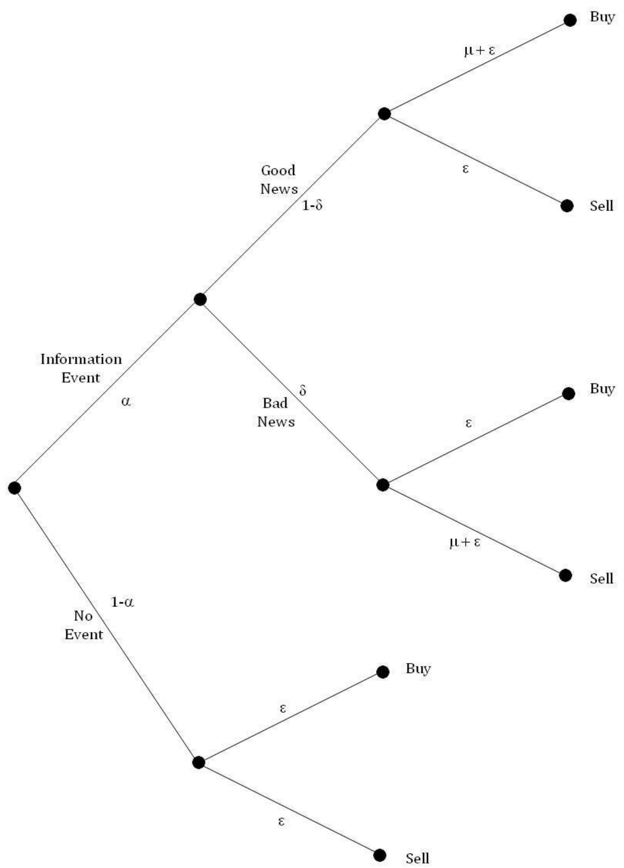


"Order flow toxicity" (measured as CDF[VPIN]) was at historically high levels one hour prior to the *flash crash*

The theory underlying this model was first peer-reviewed and published in the Journal of Finance (1996), followed by subsequent publications in the Journal of Financial Econometrics (2008), Journal of Portfolio Management (2010), Journal of Trading (2011), the Review of Financial Studies (2012) and Mathematical Finance (2013). Since then, it has been applied in a variety of settings and asset classes.

Theoretical foundations

VPIN has its origins in the seminal work of Professors Maureen O'Hara and David Easley, of Cornell University. In the year 1996, they co-authored (with N. Kiefer and J. Paperman) a study published in the Journal of Finance, which derived a magnitude known as Probability of Informed Trading (**PIN**). Using a sequential trading model with Bayesian updates, these authors proposed a Market microstructure theory to explain the range at which market makers are willing to provide liquidity. This theory was well accepted in Academic and practitioner's forums, and has since been included in most market microstructure textbooks.[1]



The sequential trading diagram of the 1996 PIN model

Summarily, denote a security's price as S. Its present value is S_0. However, once a certain amount of new information has been incorporated into the price, S will be either S_B (bad news) or S_G (good news). There is a probability α that new information will arrive within the time-frame of the analysis, and a probability δ that the news will be bad (i.e., $(1-\delta)$ that the news will be good). The authors prove that the expected value of the security's price can then be computed at time *t* as

$$\mathrm{E}[S_t] = (1-\alpha_t)S_0 + \alpha_t\left[\delta_t S_B + (1-\delta_t)S_G\right] \ .$$

Following a Poisson distribution, informed traders arrive at a rate μ, and uninformed traders at a rate ε. Then, in order to avoid losses from informed traders, market makers reach breakeven at a bid level

$$\mathrm{E}[B_t] = \mathrm{E}[S_t] - \frac{\mu\alpha_t\delta_t}{\epsilon + \mu\alpha_t\delta_t}(\mathrm{E}[S_t] - S_B) \ .$$

and the breakeven ask level at time *t* must be

$$\mathrm{E}[A_t] = \mathrm{E}[S_t] + \frac{\mu\alpha_t(1-\delta_t)}{\epsilon + \mu\alpha_t(1-\delta_t)}(S_G - \mathrm{E}[S_t]) \ .$$

It follows that the breakeven bid-ask spread is determined as

$$\mathrm{E}[A_t - B_t] = \frac{\mu\alpha_t(1-\delta_t)}{\epsilon + \mu\alpha_t(1-\delta_t)}(S_G - \mathrm{E}[S_t]) + \frac{\mu\alpha_t\delta_t}{\epsilon + \mu\alpha_t\delta_t}(\mathrm{E}[S_t] - S_B) \ .$$

For the standard case

$$\delta_t = \frac{1}{2} \Rightarrow \mathrm{E}[A_t - B_t] = \frac{\alpha_t\mu}{\alpha_t\mu + 2\epsilon}\left(S_G - S_B\right) \ .$$

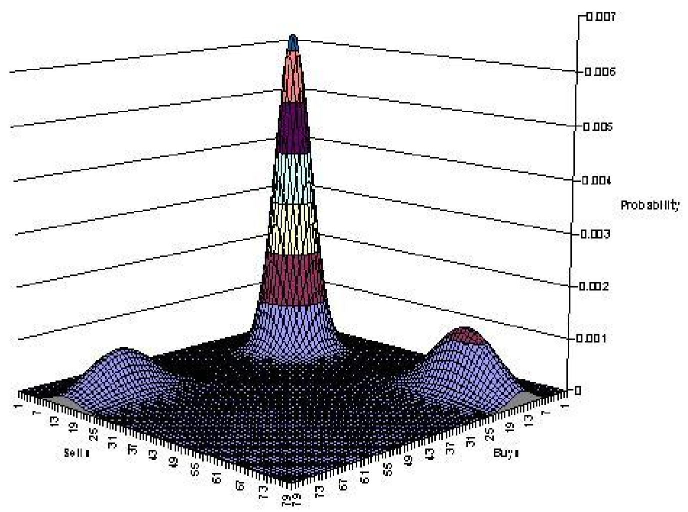
which tells us that the critical factor that determines the range at which market makers provide liquidity is

$$PIN_t = \frac{\alpha_t\mu}{\alpha_t\mu + 2\epsilon}$$

The subscript *t* indicates that probabilities α and δ are estimated at that point in time, and the authors use a Bayesian updating process to incorporate information after each trade arrives to the market.

Low-Frequency Estimation

The original **PIN** model requires the estimation of four non-observable parameters, namely α, δ, μ, and ϵ. This was originally done via Maximum likelihood, through the fitting of a mixture of three Poisson distributions,



Fitting the parameters of a PIN model through a mixture of 3 Poisson distributions

$$P[V^B,V^S]=(1-\alpha)P[V^B,\epsilon]P[V^S,\epsilon]+\alpha(\delta P[V^B,\epsilon]P[V^S,\mu+\epsilon]+(1-\delta)P[V^B,\mu+\epsilon]P[V^S,\epsilon]) .$$

where V^B is the volume traded against the Ask and V^S the volume traded against the Bid.

In a paper published in the Journal of Financial Econometrics (2008), David Easley, Robert Engle, Maureen O'Hara and Liuren Wu proposed a dynamic estimate in discrete time.[2] These authors proved that

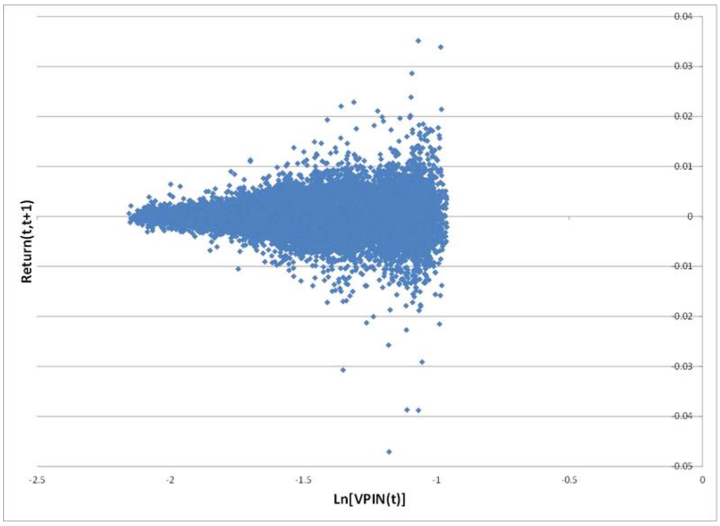
$$\mathrm{E}[V^B-V^S]=\alpha(1-\delta)(\epsilon-(\mu+\epsilon))+\alpha\delta(\mu+\epsilon-\epsilon)+(1-\alpha)(\epsilon-\epsilon)=\alpha\mu(1-2\delta) .$$

and in particular, for a sufficiently large μ

$$\mathrm{E}[|V^S-V^B|]\approx\alpha\mu .$$

High-Frequency Estimation

In the year 2010, David Easley, Marcos Lopez de Prado and Maureen O'Hara proposed a high-frequency estimate of **PIN**, which they denominated **VPIN**. This procedure adopts a *volume clock* which synchronizes the data sampling with the market activity, as captured by regular volume *buckets*.



Periods of high VPIN values (x-axis) are followed by increased volatility (y-axis)

This is a form of subordinated stochastic process that departs from the standard *chronological clock* (i.e., sampling at regular time periods), and can be traced back to the work of Benoit Mandelbrot. These authors begin by dividing a sample of volume bars (or similarly, time bars) in *volume buckets* (groups of trades such that each group contains the same amount of traded volume). Because all buckets are of the same size, V,

$$\frac{1}{n}\sum_{\tau=1}^{n}\left(V_\tau^B + V_\tau^S\right) = V = \alpha\mu + 2\epsilon \ .$$

where *n* is the number of volume bucket used to estimate **VPIN**. The procedure requires a method to split volume in *buys* and *sells*. Rather than using the *Tick-rule*, *Lee-Ready* or other trade classification techniques, they propose a new volume classification method called *Bulk Volume Classification*. This departs from standard trade classification schemes in two ways: First, volume is classified in bulk, and second this methodology classifies part of a bar's volume as *buy*, and the remainder as *sell*. Empirical studies have shown *Bulk Volume Classification* to be more accurate than the *Tick-rule*, despite of not requiring level-1 tick data (only bars).[3] Within a volume bucket, the amount of volume classified as buy is

$$V_\tau^B = \sum_{i=t(\tau-1)+1}^{t(\tau)} V_i Z\left(\frac{S_i - S_{i-1}}{\sigma_{\Delta S}}\right) \ .$$

where $t(\tau)$ is the index of the last (volume or time) bar included in bucket τ, V_τ^B is the *buy volume* (traded against the Ask), V_i is the total volume per bucket, Z is the Standard normal distribution, and $\sigma_{\Delta S}$ is the standard deviation of price changes between (volume or time) bars. Because all buckets contain the same amount of volume V,

$$V_\tau^S = \sum_{i=t(\tau-1)+1}^{t\tau} V_i\left(1 - Z\left(\frac{S_i - S_{i-1}}{\sigma_{\Delta S}}\right)\right) = V - V_\tau^B \ .$$

Since $\mathrm{E}[|V^S - V^B|] \approx \alpha\mu$, and $PIN = \dfrac{\alpha\mu}{\alpha\mu + 2\epsilon}$, it can be shown that VPIN is a good estimate of PIN, with

$$VPIN = \frac{\sum_{\tau=1}^{n}|V_\tau^S - V_\tau^B|}{nV} \ .$$

Applications

PIN and VPIN have been found useful in a number of settings. As a result, applications of this theory have been the subject of three international patent applications.

VPIN and liquidity crashes

Video of the S&P500 futures during the *Flash Crash* [4]

Toxicity-induced volatility on August 4, 2011 [5]

Toxicity-induced volatility in Energy markets [6]

This theory has been used to monitor the stress to which Market makers are subjected by informed traders, thus providing a high-frequency metric of the probability that the liquidity provision process may fail. This applies to liquidity crises such as the 2010 Flash Crash. On May 6, 2010, one hour before its collapse, the stock market registered some of the highest readings of *order flow toxicity* in recent history. The authors of this paper applied widely accepted Market microstructure models to understand the behavior of prices in the minutes and hours prior to the crash. According to this paper, *order flow toxicity* can be measured as the probability that informed traders (e.g., hedge funds) adversely select uninformed traders (e.g., Market makers). For that purpose, they developed the VPIN Flow Toxicity metric, which delivers a real-time estimate of the conditions under which liquidity is being provided. If the order flow becomes too toxic, market makers are forced out of the market. As they withdraw, liquidity disappears, which increases even more the concentration of toxic flow in the overall volume, which triggers a Feedback mechanism that forces even more market makers out. This cascading effect has caused hundreds of liquidity-induced crashes in past, the *flash crash* being one (major) example of it. One hour before the *flash crash*, order flow toxicity was at historically high levels relative to recent history.

VPIN's theory is consistent with the anecdotal evidence reported by the joint SEC-CFTC study on the events of May 6, 2010. Given the relevance of these findings, the S.E.C. requested an independent study to be carried out by the Lawrence Berkeley National Laboratory. This Government laboratory concluded:[7]

> *This [VPIN] is the strongest early warning signal known to us at this time.*

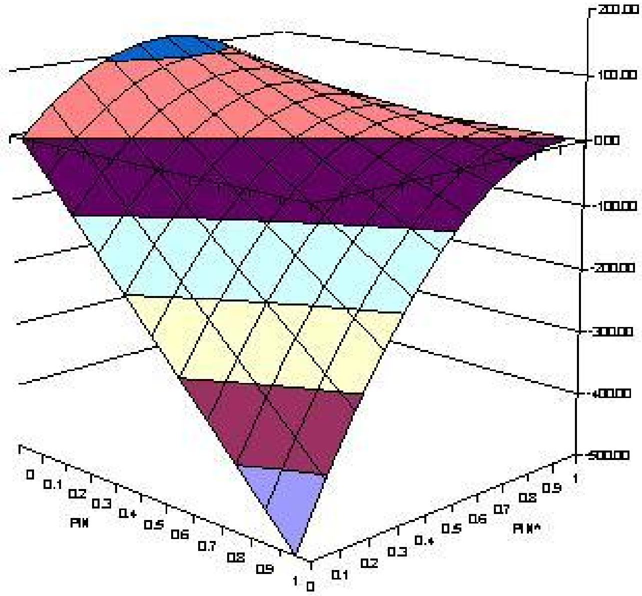
Far from being an exception, these studies show that liquidity crises are becoming more prevalent as High-frequency trading strategies are dominating the market-making process.

A second independent study confirmed VPIN's forecasting power when predicting toxicity-induced volatility on cash stocks.[8]

> *Although VPIN metric is conceived for the HFT environment, our results suggest that certain VPIN specifications provide proxies for adverse selection risk similar to those obtained by the PIN model. Thus, we consider that the key variable in the VPIN procedure is the number of buckets used and that VPIN can be a helpful device which is not exclusively applicable to the HFT world.*

Clive Corcoran's book *Systemic Liquidity Risk and Bipolar Markets: Wealth Management in Today's Macro Risk On / Risk Off Financial Environment* contains a chapter titled *Detecting mini bubbles with the VPIN metric*, in which this author demonstrates how the VPIN theory can be used to monitor the probability of toxicity-induced liquidity crises.[9]

Protection against adverse selection



The dilemma faced by market makers when proving liquidity under asymmetric information. This surface shows the profit associated with the correct estimation of PIN, and the much greater losses that result from its underestimation

In order to preserve the integrity of the liquidity provision process, two solutions have been proposed.

The first one involves a futures contract that would offer market makers protection against a rise in the probability of adverse selection.[10] Suppose that a market maker is willing to bid the market at a level B^*, and offer at a level A^*. Market makers do this with passive orders, whereby they do not choose the timing of the execution, thus making them vulnerable to the phenomenon of Adverse selection. Consequently, market makers are sellers of an implied option to be adversely selected, at a premiun of $\frac{A^* - B^*}{2}$. It has been shown that their profit (or loss) is a function of how accurately they have estimated the actual value of PIN,

$$Profit = \frac{(S_G - S_B)\,\kappa}{2}(1 - PIN^*)(PIN^* - PIN) \; .$$

where κ is a constant that relates the volume traded to the range at which liquidity is provided,

$$V = \kappa\left(1 - \frac{A^* - B^*}{S_G - S_B}\right) \; .$$

Because the losses associated with underestimating PIN are so much greater than the potential profit when it is correctly estimated, market makers have incentives to be extremely conservative and liquidate their inventory as soon as they perceive the presence of informed traders. This situation is detrimental to the market, and can derive in serious liquidty crises such as the 2010 Flash Crash. As a solution, a contract could be issued to offer protection against adverse selection. Market makers would buy protection when they see their inventory rising beyond normal levels. On the other hand, informed traders would be interested in selling that protection once their orders have been completed, thus monetizing on their private knowledge that the portion of toxicity they were responsible for will cease.

A second solution consists in dynamically adjusting the speed of the matching engine. If the bids are being hit at such speed that market makers have no chance to replenish liquidity, market makers will be forced out and a liquidity crash will occur. An alternative would be, under such circumstances, slowing down the speed at which matches occur at the bid, while speeding up the matches that occur at the ask. This two-speed solution (also known as *yellow flag*) differs from the circuit breaker (or *red flag*) approach currently in place. While the *red flag* approach stops the market after the crisis has unfolded, the *yellow* flag would try to avoid the crisis in the first place, thus allowing exchange's activity to proceed uninterrupted.

VPIN and volatility

The purpose of the VPIN theory is understanding how toxicity is a source of volatility. VPIN is not a volatility forecasting model. However, one of the several reasons why volatility may occur is as a result of market makers widening their trading ranges. This is a particular form of volatility, which is induced by increased levels of order-flow toxicity. Empirical evidence seems to corroborate that VPIN can help predict toxicity-induced volatility through machine learning algorithms.

VPIN and execution

A large order reveals information to other market participants, who may take advantage of that information by frontrunning that order before its completion. The purpose of an *Optimal Execution Horizon* (OEH) model is to compute the trading horizon that minimizes that informational leakage without incurring in unnecessary market risk. The length of that optimal horizon depends of factors such as: The size of the order, the side, the prevalent order imbalance, market volatility, the trading range and risk aversion. From a theoretical perspective, OEH explains why market participants may rationally 'dump' their orders in an increasingly illiquid market. OEH has been shown to perform better than participation rate schemes. This model is derived as follows:

We have seen earlier that when $\delta_t = \frac{1}{2}$, we obtain that

$$\mathrm{E}[A_t - B_t] = \frac{\alpha_t \mu}{\alpha_t \mu + 2\epsilon}\left(S_G - S_B\right) \approx \frac{\mathrm{E}[|V^S - V^B|]}{V}\left(S_G - S_B\right)\ .$$

This means that we would like to compute V such that $\frac{\mathrm{E}[|V^S - V^B|]}{V}$ is minimally impacted. First, ceteris paribus, the impact of an order m on the order imbalance over the next bucket V is

$$\frac{|V^S - V^B|}{V} = |\left(2v^B - 1\right)\left(1 - \frac{|m|}{V}\right) + \frac{m}{V}|\ .$$

where $v^B = \frac{V^B}{V}$ and $\left(2v^B - 1\right)$ is the order imbalance prior to m. Let $\phi\left[|m|\right]$ be a monotonic increasing function of $|m|$, $0 < \phi\left[|m|\right] < 1$, which measures the displacement that $|m|$ causes to the previous order imbalance. Then, the new persistent order imbalance can be computed as

$$\tilde{OI} = \phi\left[|m|\right]\left[\left(2v^B - 1\right)\left(1 - \frac{|m|}{V}\right) + \frac{m}{V}\right] + \left(1 - \phi\left[|m|\right]\right)\left(2v^B - 1\right)\ .$$

Second, assuming that prices follow an arithmetic random walk, for a risk aversion λ we can derive a timing risk

$$P\left[sgn\left(m\right)\Delta S > Z_\lambda \sigma\left(\frac{V}{V_\sigma}\right)^{\frac{1}{2}}\right] = 1 - \lambda\ .$$

Then, a *probabilistic loss function* can be defined as the aggregation of the first (liquidity risk) and second (timing risk) components,

$$\Pi\left(V\right) = |\phi\left[|m|\right]\left[\left(2v^B - 1\right)\left(1 - \frac{|m|}{V}\right) + \frac{m}{V}\right] + \left(1 - \phi\left[|m|\right]\right)\left(2v^B - 1\right)|\left(S_G - S_B\right) - Z_\lambda \sigma\left(\frac{V}{V_\sigma}\right)^{\frac{1}{2}}\ .$$

Finally, $\Pi\left(V\right)$ can be minimized with respect to V. A novel feature of this model is that, beyond the order size, other important variables are used in determining the optimal V, like the side of the order, and the prevalent order imbalance.

Federal oversight of Financial Markets

In a peer-reviewed paper published in the Journal of Trading,[11] scientists at the Lawrence Berkeley National Laboratory showed that VPIN would be a useful metric to monitor in real time the probability of a liquidity crisis. According to the Wall Street Journal:[12]

> *The SEC has estimated that a centralized order-tracking system would cost approximately $4 billion to set up and $2.1 billion a year to maintain. Mr. Leinweber of Berkeley has a simpler, and probably cheaper, solution in mind. He proposes that supercomputers—like those at national laboratories such as Berkeley's—should track every trade in real time. If volume began surging dangerously, the system would flash a "yellow light." Regulators or stock exchanges could then slow trading down, giving the market time to clear and potentially averting a crisis.*
>
> *By monitoring spikes in trading, the formula [VPIN] may offer early warning that a particular security—or an entire market—is about to be overwhelmed with buy or sell orders... An SEC official says the agency is aware of this research and regards it as "interesting," but that the data can't be analyzed until someone figures out how to get all of it in one place.*

Regarding the use of VPIN to monitor financial markets, Dr. Horst Simon, Deputy Director of the Lawrence Berkeley National Laboratory remarked:

> *If we can help the financial markets to be more reliable and stable, then that's at least as important a national need. Mr. Simon worries it might take some kind of market catastrophe "for people to wake up and say that there's a real danger out there of our whole system being brought down by a simple [problem] that could have been prevented if we had just paid attention.*

Professors of the Tinbergen Institute and VU University Amsterdam have examined VPIN during the flash crash, and show that "the large seller's relative presence in the market co-moves negatively with flow toxicity. This finding is consistent with strategic trading: she sells passively during upturns (her limit sell orders are taken out), sells aggressively right after an upturn, and does not trade in downturns." [13] This analysis confirms previous studies supportive of VPIN as a useful indicator of order flow toxicity.[14]

Implementation cautions

A few mistakes can lead to incorrect implementations of VPIN. First, VPIN's estimates are based on (volume or time) bars, not tick (or level 1 transactional) data. Second, VPIN applies the *Bulk Classification* algorithm, not the *Tick-rule* for volume classification. Third, $Ln\left(VPIN_{\tau-1}\right)$, and not $VPIN_{\tau-1}$, can be used to forecast $\frac{S_\tau}{S_{\tau-1}} - 1$. Fourth, the correlation between $Ln\left(VPIN_{\tau-1}\right)$ and $\frac{S_\tau}{S_{\tau-1}} - 1$ is significantly high, however the authors recommend using a conditional probability approach, as opposed to traditional regression techniques. Alternative implementations of VPIN will yield results inconsistent with this theory.

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External links

- Flow Toxicity and Volatility (http://ssrn.com/abstract=1695596), David Easley (Cornell University), Marcos López de Prado (Tudor Investment Corp., RCC at Harvard University) and Maureen O'Hara (Cornell University), Review of Financial Studies, forthcoming.
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